

An Improved Body Shape Definition for Acoustic Guitars

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Abstract—Modern guitar design and manufacturing processes require a robust mathematical description of body shapes. Conventional curve fits such as those offered in commercial software are not possible because the shape of guitar bodies creates mathematical problems. An attractive solution is to select a reference point on the centerline of the guitar and do the curve fit in polar coordinates. This solves the mathematical problems and can give a closed form expression to become the basis of a family of body shapes.

I. BACKGROUND

There are a small number of commonly-used acoustic guitar body shapes. However, there don't appear to be clear, compact mathematical descriptions of them. If guitars are to be made using computer controlled tools such as CNC routers, it makes sense to establish compact, unambiguous definitions of body shapes.

Figure 1 shows a family of acoustic guitar design from Bourgeois Guitars (bourgeoisguitars.net). Naming conventions for acoustic body shapes are, like much else in the guitar world, not standardized. Probably the most common terminology stems from models produced by Martin and is reflected in the Bourgeois designs.

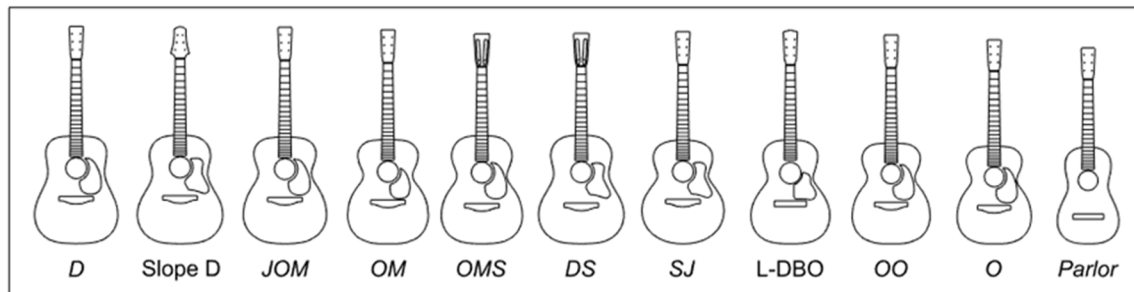


Figure 1 – Some Common Guitar Body Shapes (bourgeoisguitars.net, image used with permission)

It is perhaps worth noting that some manufacturers, including Taylor Guitars, use a naming convention that includes, in order of increasing size: mini, parlor, grand concert, grand auditorium, dreadnaught, jumbo. These names are based on Martin product names rather than product designations. That said, the method presented here is general and doesn't limit the user to one body shape or another.

In a world where computer-controlled mills are used to make guitars in environments ranging from large factories to home shops, it is especially important to have precise, mathematical definitions of body shapes. Small builders often use full size plans that are, in turn, copied from existing instruments [1]. Alternatively, some luthiers work from proportions developed from groups of existing guitars. **Figure 2** shows proportions for classical guitars as suggested by Richard Bruné [2].

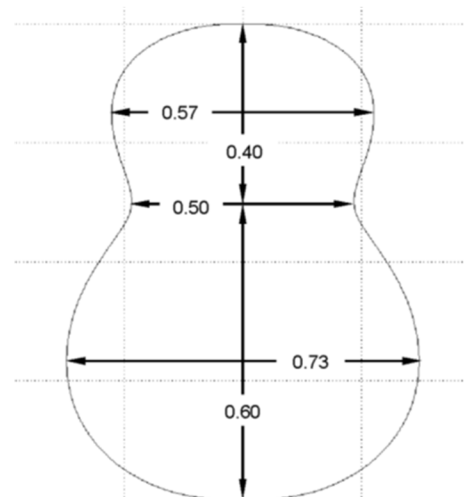


Figure 2 – Classical Guitar Proportions

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The next step beyond working from drawings or lists of a few body proportions is to calculate shapes based on groups of intersecting curves. A well-executed example is a program called (somewhat inexplicably) “G” Thang that can develop a wide range of body shapes from a list of user-specified parameters [3]. **Figure 3** shows an example modeled after the OM body.

While these approaches certainly work, they may not be enough for a precise manufacturing operation.

For manufacturing, there must be clear, unambiguous mathematical definitions for all components. Mathematical definitions of body shapes pretty much take one of three forms:

- 1) Drawings based on a few basic dimensions
- 2) Collections of curves (splines) based on drawings or basic dimensions
- 3) Closed form expressions

We’re all familiar with the idea of building from drawings and many of us have made guitars using drawings on butcher paper. Progressing to Computer Aided Design (CAD) software, it is easy to define complex curves using a mathematical tool called splines. These are basically a succession of short, simple curves that are stitched together to make the final shape. For example, AutoCAD uses a form called Non-uniform Rational B-Splines or NURBS, for short (not the best name, perhaps, but there it is). In practice, NURBS are often based on cubic polynomials [4].

Splines have the advantage that they are mathematically precise and unambiguous. However, it can be awkward to move spline data around. Rather than a few coefficients that might be used to define a function, splines require a data file containing lists of coefficients. There is also always the risk that different software packages might store and import spline data differently. Finally, having a file of spline data is of no use unless you also have software that will import and correctly interpret it.

Using splines may give you a manufacturing process that is dependent on decisions you’ve made about CAD software. To be fair, there are portable file formats that can be imported into most CAD software. The problem remains, though, that using a collection of splines means that you don’t have a closed form expression for the shape of your own design.

An attractive alternative is to use a closed form mathematical expression – one that can be simply written out and evaluated using some kind of generic calculation software like Excel, MATLAB or OCTAVE. Once you’ve selected a function to use, it is independent of whatever software you wish to use and it can be written out completely in a very compact form. Thus, it is easy to share and hard to misinterpret.

II. INTRODUCTION

Ideally, the body description could be generated easily from a short list of coefficients and a single closed-form function. The only problem in defining the body shape is that the typical guitar body shape is mathematically challenging. For this project, I chose a small number of points measured from a guitar and modified a little to suit my own aesthetic preferences. The next step was to do a curve fit to find an expression that fit those points and didn’t have any undesirable wiggles in it. The work presented here is an improved implementation of an approach published earlier [5]. **Figure 4** shows the points I used.

When not using splines, there are two rules for curve fits. The first is that the curve, $y(x)$, can have only one y value for every x value. Mathematicians call this a single valued function. In practice, it means that the function cannot double back on itself. The second rule is that the closed form expressions can’t have vertical slopes.

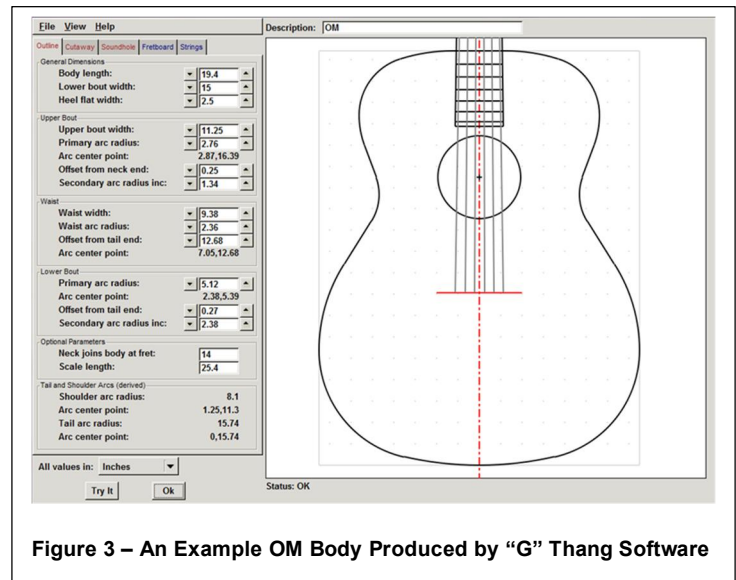


Figure 3 – An Example OM Body Produced by “G” Thang Software

So there is clearly a problem in trying to develop a curve for a guitar body as shown in Figure 4; the slopes are vertical at both ends of the body. Rotating the body to be vertical makes the problem worse since there will be a vertical slope at the waist and both bouts, and there will be more than one point for most values of x. Fortunately, there is a way to fix both problems.

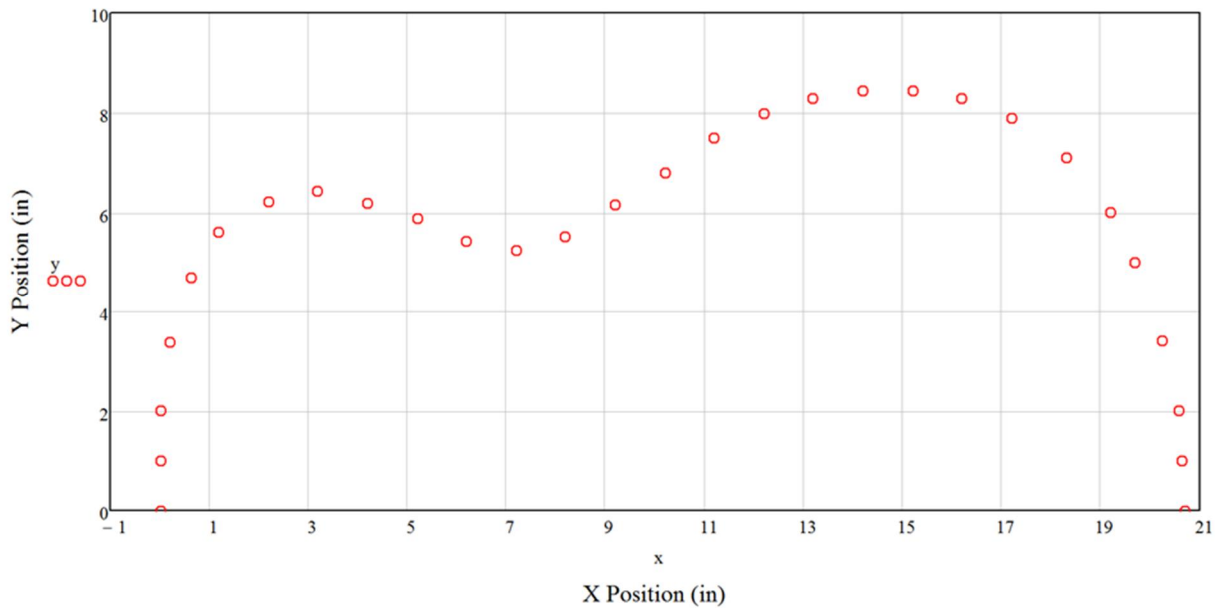
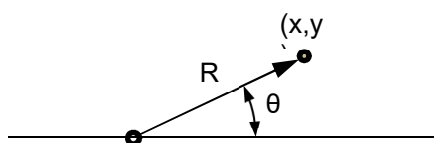


Figure 4 – Points Chosen for Guitar Body

III. DEVELOPMENT OF POLAR CURVE FIT

Engineering students spend time in their various classes learning how to transform problems they can't solve into equivalent problems they can solve. This is one of those times. The solution here is to transform the problem into polar coordinates so that each point is described by an angle and a radius from some reference point. Any coordinate can be expressed in rectangular or polar coordinate and it is easy to convert from one to the other.



$$R = \sqrt{x^2 + y^2} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = R \cos(\theta) \quad y = R \sin(\theta)$$

For this body shape, I used x=9, y=0 as the reference point (origin of the coordinate system) so angles are calculated with respect to that point. Now, the points are mathematically well-behaved when plotted in rectangular coordinates, as shown in **Figure 5**.

Fitting the points is now a much more tractable problem and the process progresses in two steps. The first is to get close to the points with a general purpose function and the second is to add some refinements to make sure resulting function is practical for making guitars.

The most useful class of general functions I've been able to identify is rational polynomials. These are ratios of two polynomials. In a previous effort, I had to use a function made of two 9th order polynomials (9/9 order). While it worked well enough, this was an awkwardly complex function and it had some small scale wiggles that were obvious when using a CNC router to make forms. For this improved shape, I used a simpler 5/5 order function with three small correction terms. The parameters of the 5/5 function were identified using the MATLAB curve fitting tool [6].

$$y(\theta) = \frac{p_1\theta^5 + p_2\theta^4 + p_3\theta^3 + p_4\theta^2 + p_5\theta + p_6}{\theta^5 + q_1\theta^4 + q_2\theta^3 + q_3\theta^2 + q_4\theta + q_5} \tag{1}$$

Figure 6 shows the 5/5 order curve fit superimposed over the points used to create it. At first glance, it looks pretty good, perhaps good enough to be used for making guitars. However, a closer look shows that there are lingering differences that can cause practical problems when trying to make a guitar body.

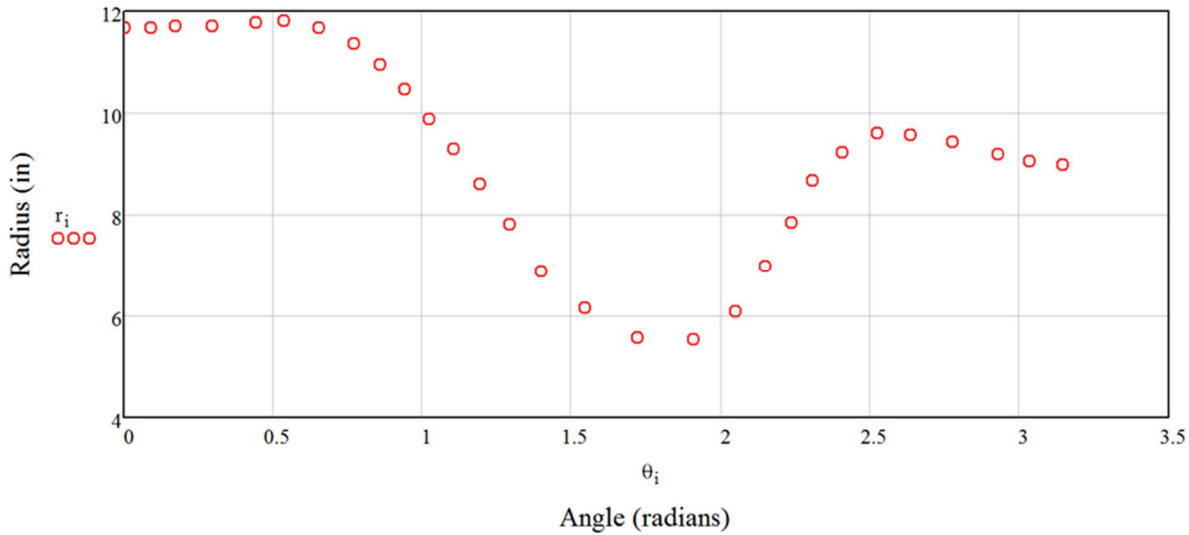


Figure 5 – Polar Coordinates from Guitar Body Plotted on Rectangular Axes

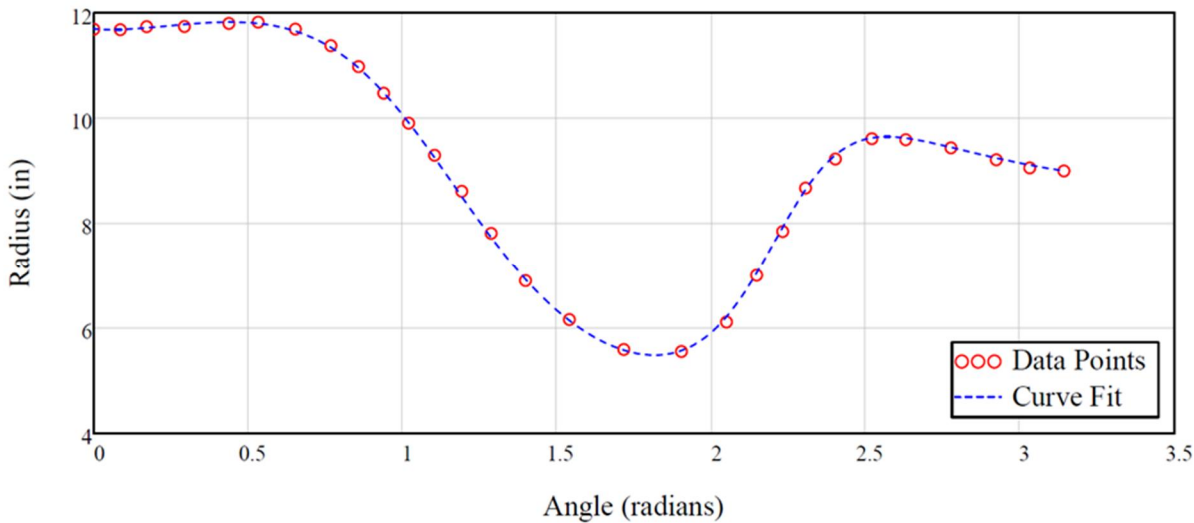


Figure 6 – A 5/5 Order Curve Fit Superimposed over Points Used to Create It

The first source of error to be address is a small error that grows as θ increases. The error doesn't grow smoothly, but a smooth correction function fixes most of the problem. After a few attempts, the most useful correction function was

$$\Delta_1(\theta) = a_1 \sin(a_2 \theta) \sqrt{\theta} \tag{2}$$

Another problem was that the function describing the body form is not vertical where the neck would connect, as shown in **Figure 7**. This is where θ is close to π (3.14159 radians).

Even though the scale in **Figure 7** is much expanded, it's clear that this shape isn't acceptable when trying to fit the neck accurately to the body. The simplest way to fix the problem is to add a second correction term. This one needs to be close to zero everywhere except where θ is close to π (close to 180°). A good correction term is

$$\Delta_2(\theta) = b_1 e^{b_2(\theta - b_3)} \tag{3}$$

This is basically zero everywhere except the region close to the neck joint as shown in **Figure 8**.

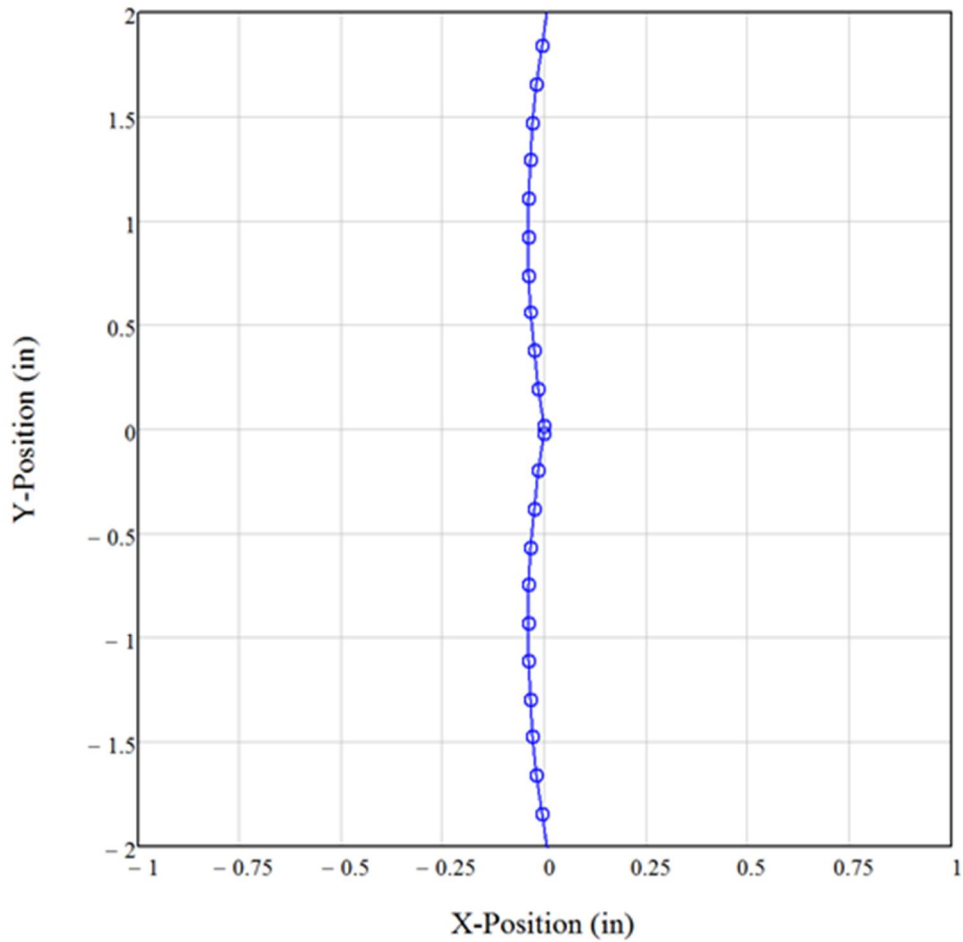


Figure 7 – Uncorrected Body Shape at Neck Joint

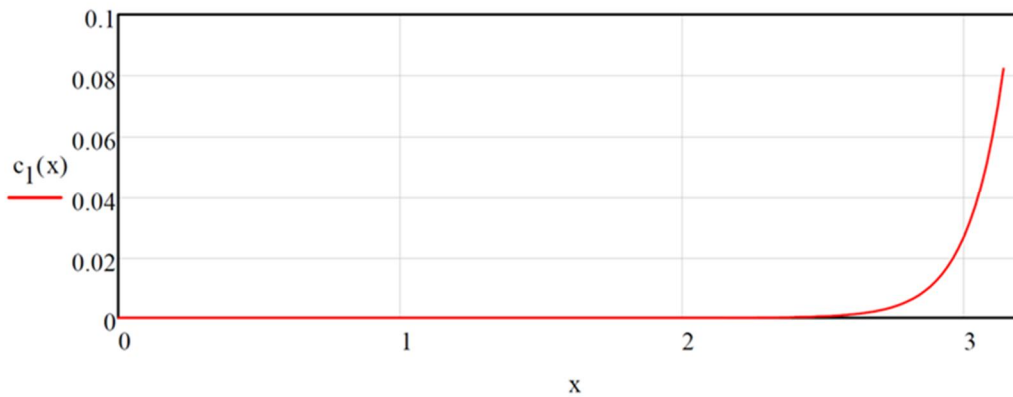


Figure 8 – Second Shape Correction Function

The body shape is now flat and vertical at the neck joint, as shown in **Figure 9**. Note that the correction function (red diamonds) is magnified by a factor of 2 in order to make it easier to see.

A similar correction function makes sure that the slope at the tail end of the body is also vertical.

$$\Delta_3(\theta) = c_1 e^{-c_2 \theta} \tag{4}$$

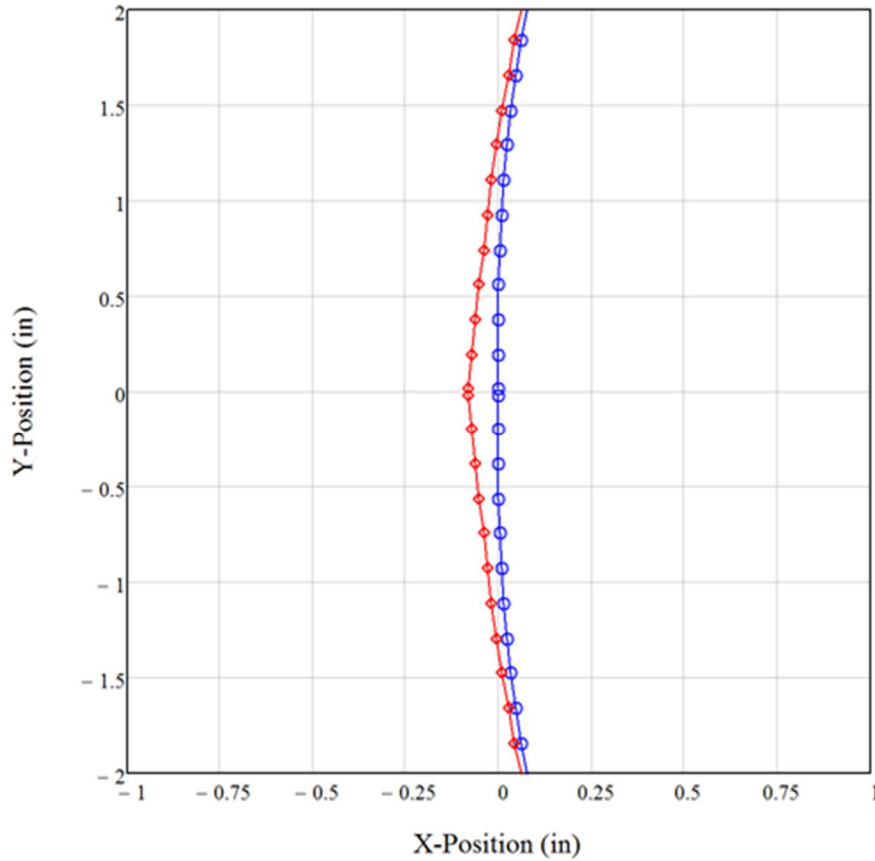


Figure 9 – Result of Neck Correction

IV. RESULTING BODY SHAPE

Figure 10 shows the resulting body shape, with all three correction terms. It is largely free of the small scale variations that can cause problems with CNC machining.

The final body shape, with correction terms is

$$y(\theta) = \frac{p_1\theta^5 + p_2\theta^4 + p_3\theta^3 + p_4\theta^2 + p_5\theta + p_6}{\theta^5 + q_1\theta^4 + q_2\theta^3 + q_3\theta^2 + q_4\theta + p_5} + a_1 \sin(a_2\theta) \sqrt{\theta} + b_1 e^{b_2(\theta - b_3)} + c_1 e^{-c_2\theta} \quad (5)$$

and the parameters are listed in Table 1.

This design is about as big as guitar bodies get. The body length is 20.7 inches and lower bout width is 17 inches. However, it is easy to scale the x and y dimensions to give the size you want. Figure 10 shows the original body shape along with the shape scaled for a parlor guitar and a 3/4 size travel guitar – approximately the body dimensions of a Taylor GS Mini. Note that the points developed in polar coordinates using Equation (5) were converted to rectangular coordinates for Figure 11. Another option is to divide the list of (x,y) coordinates for the body by 20.7 so that the body length is 1. That way, you can scale the body by simply multiplying the coordinates by the desired body length.

Parameter	Value
p ₁	8.933
p ₂	-60.03
p ₃	151.9
p ₄	-
	167.31
p ₅	54.5
p ₆	23.51
q ₁	-6.449
q ₂	15.26
q ₃	-15.45
q ₄	4.772
q ₅	2.01
a ₁	0.04
a ₂	2
b ₁	80
b ₂	8
b ₃	4
c ₁	-0.08
c ₂	-8

Table 1 – Parameters for Body Curve Fit

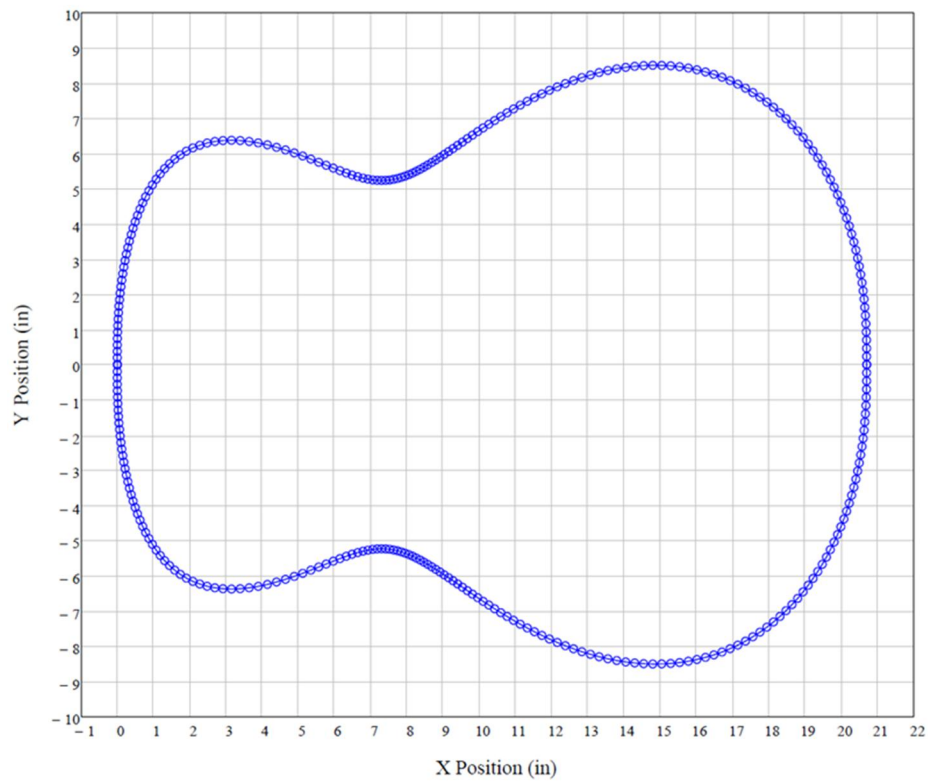


Figure 10 – Final Body Shape

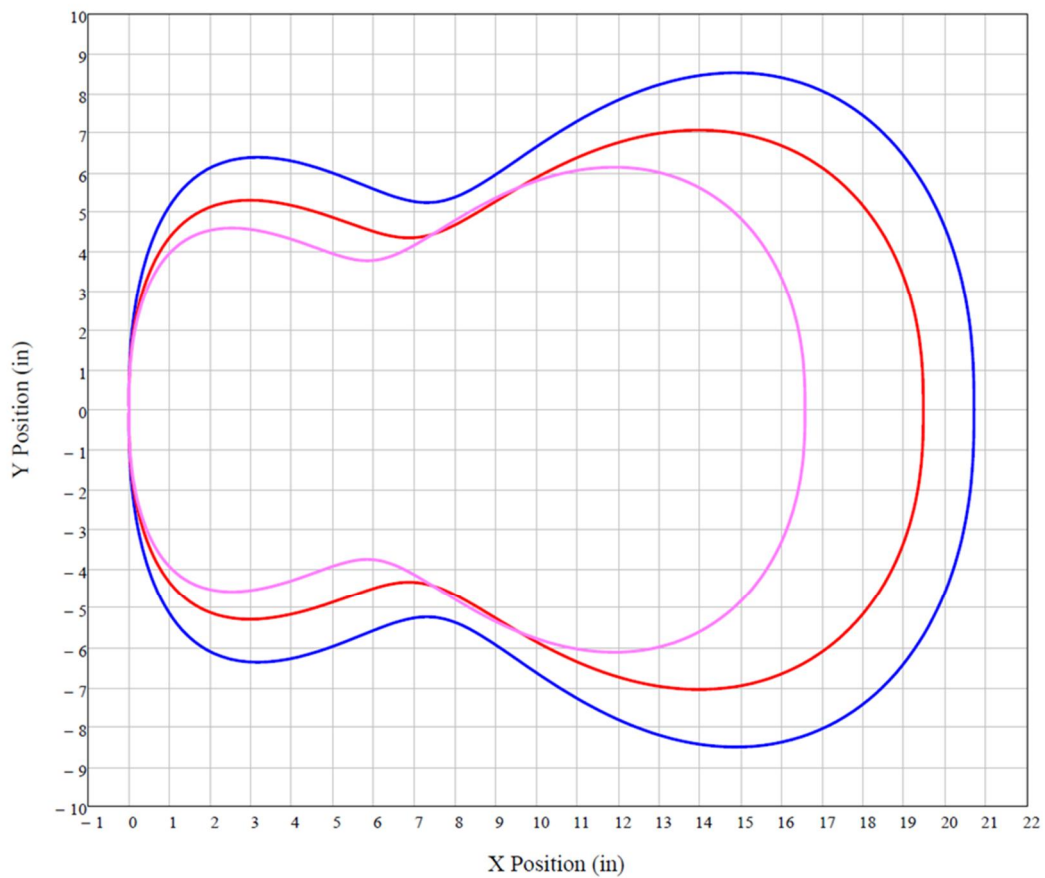


Figure 11 – Producing Different Size Bodies by Scaling the Basic Shape

Table 2 shows the scaling factors used to produce the shapes in **Figure 11**. A family of guitars based on these body shapes is being developed. The early focus will be on the parlor and $\frac{3}{4}$ shapes, though a ukulele may be added.

Name	Body Length	Lower Bout Width	X Scale Factor	Y Scale Factor
Original	20.7	17	1	1
Parlor	19.5	14.1	0.94	0.83
$\frac{3}{4}$ Acoustic	16.5	12.25	0.8	0.72

Table 2 – Scale Factors for Different Body Shapes (dimensions in inches)

V. BIBLIOGRAPHY

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