Detection of Bow Direction and Stopped String Length by Analysis of Asymmetric Helmholtz Velocity Data from a Magnetic Violin Pickup

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Abstract—The strings of bowed musical instruments undergo "Helmholtz motion" and exhibit time-varying displacements and velocities that contain information about the length of the string and the direction of action of the forces of static friction from the bow hair. This information is largely obscured in the sound radiated from acoustic instruments and in the output of pickups located on the bridge or instrument body. However, pickups that directly sense string dynamics can recover these information-rich signals from the bowed string. From the asymmetry of Helmholtz motion it is possible to deduce from pickup signal waveforms both the direction of the bow's motion (up-bow or down-bow) and the approximate length of the vibrating string as these change during a performance. Computationally simple methods applied to the output of a magnetic pickup permit recovery of aspects of a player's bowing and left hand technique for study, or in electronic music, the control of performance parameters by bow direction or left hand position.

I. INTRODUCTION

In Appendix VI to his classic treatise "On the Perception of Tone" [1], the renowned physician and physicist Hermann von Helmholtz set forth the equations of motion of musical instrument strings during bowing. When driven with a bow, the displacement of a small segment of the vibrating string is characterized by a generalized ramp wave having two distinct velocities. These velocities are equal at the midpoint of the string, so that the displacement is a symmetric triangle wave at that position. Nearer the nut or bridge, the two velocities become increasingly different, resulting in an increasingly asymmetric ramp waveform. At the extreme ends of the string the displacement approaches a sawtooth wave.

While the acoustic vibrations of the instruments of the violin family are driven by the sawtooth waveform of string forces at the bridge, the compliance of the wooden instrument body largely obscures the sawtooth character of the driving forces in the radiated sound. However, transducers or pickups which sense string motions directly may enable analysis of the asymmetric displacements of the string in real time. In particular, this paper describes methods whereby the changing length of a bowed string, and changes in bow direction (up-bow versus down-bow playing) may be deduced by near real-time analysis of signals from a magnetic pickup. These analyses offer the ability to recover additional information about a player's technique (fingering, bowing, etc.) that is not available in a conventional recording.

II. HELMHOLTZ MOTION AND THE BOWED STRING

Throughout this paper, we will adopt the nomenclature of Helmholtz [1] to describe the time-dependent displacement $y(t)$ of a small region of a string from its equilibrium position during driving by the rosin-coated horsehair of a bow. This displacement (Figure 1) is a generalized ramp wave of period T, characterized by a constant initial velocity f lasting from the commencement of a cycle of vibration at time = 0 to a later time τ. Over the remainder of the vibrational period ($T - \tau$), the string returns to its starting position at a constant velocity g.

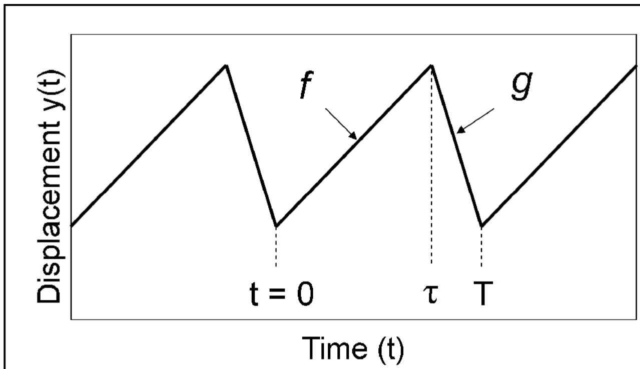

Figure 1—The displacement of a bowed string is a generalized ramp wave with period T. The rising and falling slopes, f and g vary with position (x) along the length (L) of the string.

The instruments of the violin family are traditionally played with the bow hair quite near the bridge in order to accommodate stopping of the string by the player.

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In the modern violin, for example, the bow is usually placed at a distance (x) that is 2-3 centimeters from the bridge, the string's overall length (L) being about 32 cm, so that the ratio (x/L) is approximately 0.10 to 0.06. To drive musically stable vibrations of the string at a point so near the bridge, the interaction of the bow hair and instrument string must drive the string with a strongly asymmetric sawtooth displacement. This asymmetric waveform results from the phenomenon often described as "sticking and slipping" [1,2]: a relatively long period of low velocity while the string is adherent to the bow hair and being displaced away from its equilibrium position under the influence of static friction; and a shorter period of higher velocity during which the string returns toward its equilibrium position after restoring forces overcome static friction. This characteristic "Helmholtz motion" has been studied extensively: see for example reference [3].

A necessary condition for periodicity of the generalized ramp function is that $ft = g(T - t)$, so that

$$\frac{t}{T} = \frac{g}{f+g}. \qquad (1)$$

Helmholtz [1] showed that the velocities f and g are constrained to vary at position x along the string according to

$$g = \frac{8Vx}{LT} \qquad (2)$$

and

$$f = \frac{8V(L-x)}{LT} \qquad (3)$$

where V is the maximum displacement of the string at its midpoint. By substitution, the displacement at any normalized position (x/L) along the string must therefore obey

$$\frac{t}{T} = \frac{x}{L}. \qquad (4)$$

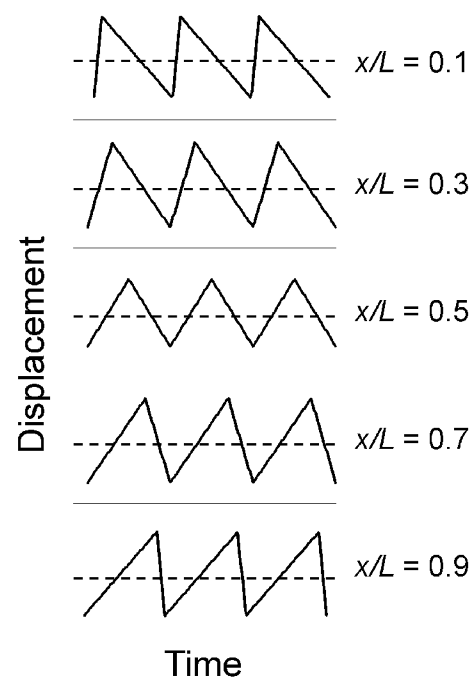

Figure 2—Displacement waveforms of the bowed string (calculated using equations (6), (9), and (10)) vary with normalized distance along the string (x/L), becoming increasingly asymmetric near the ends of the string.

Equation (4) dictates the allowable ratio (τ /T) for stable solutions to the string's equations of motion, and has been used to describe string displacements and driving forces acting at the point (x/L) where the bow contacts the string [3]. In this study, however, we will consider instead a pickup sensing string motions at a location (x/L) which might be different than the location of bow contact. In this case the displacements sensed by the pickup are given by equation (4) as shown in Figure 2.

III. Bow Direction and String Stopping

The player of a bowed instrument may opt to play any given note "up-bow" (the bow hand moving closer to the string) or "down-bow" (the player's bow hand moving away from the string). When the bow changes direction, one result is that the velocities f and g in Figure 1 are effectively interchanged. This occurs because displacement in the direction that had previously been driven by static friction and the bow's movement will, after the bow direction change, be driven by the string's restoring forces and dynamic friction. Likewise, displacement of the string in the direction that had previously been driven by the string's restoring forces now becomes driven by the bow motion and static friction. By equations (1) and (4), the interchange of velocities f and g cause the apparent ratio (x/L) at any point on the string to change to (1 – x/L) each time the bow reverses direction. This apparent change in (x/L) is not detectable at the midpoint of the string, where (x/L) = 0.5, and the displacements are symmetric (Figure 2), but when displacements are sensed near the instrument bridge the waveform is strongly influenced by bow direction. By relating it to change in bow direction, we are using Helmholtz's (x/L) ratio in a rather different way than originally intended. Obviously, the actual location of the measured string displacement does not change when the bow changes direction. Instead we are simply exploiting the aliasing of (1 – x/L) and (x/L) when the string's driving direction changes.

The ratio (*x/L)* is also influenced by changes in the string length caused by the fingers of the player stopping the string. In this case, *x* remains constant (equal to the distance from the bridge to the pickup), while *L* varies as $2^{(-n/12)}$ where n is the number of half steps above the note to which the open string is tuned. For a violin with a scale length of 32.5 cm, and a pickup mounted 2.7 cm away from the bridge, the ratio *x/L* (Figure 3) varies nonlinearly from a minimum of 0.08 for the open string to 0.31 for a note two octaves higher. The dependence shown in Figure 3 ought, then, to permit us to estimate the effective string length L if the ratio (x/L) can be deduced from the instrument's waveform. This is useful because the skilled player of a violin is free to play a particular musical pitch on any of the instrument's strings, either for purposes of ergonomic convenience or musical purpose. The result is that the same musical note may be played using as many as four substantially different values of string length L (each value corresponding to one of the four strings of the instrument). Particularly in virtuosic playing, the player's choice of string can represent valuable information about the player's technique, so that an ability to determine an apparent Helmholtz's ratio (x/L) (or the equivalent time ratio *τ/T*) can enable simultaneous deduction of the player's bow direction and string choice at any point in a performance. We will next explore how this ratio might be recovered from real-time or recorded string dynamics data.

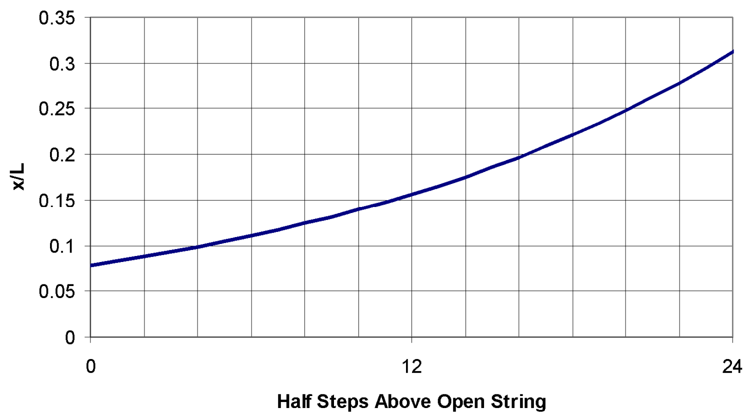

Figure 3: Calculated variation of the ratio *x/L* as a function of the note being played on a violin having scale length 32.5 centimeters and a pickup sensing string dynamics at a location (x) 2.7 centimeters away from the bridge.

IV. Fourier Analysis of Helmholtz Motion

As shown by Helmholtz (equation 3c in [1]), the displacement (y) at any point (x) along the length (L) of the string may be expressed as a Fourier series:

$$y(t) = \frac{8V}{p^2} \sum_{n=1}^{\infty} \left[\frac{1}{n^2} \sin \frac{npx}{L} \sin\left(\frac{2pn}{T}\left(t - \frac{t}{2} \right)\right)\right]. \tag{5}$$

From Helmholtz's equation and the angle addition theorem, we may eliminate the phase term in (5) to obtain an equivalent conventional Fourier expansion in terms of sines and cosines as:

$$y(t) = \sum_{n=1}^{\infty} a_n \cos\left(\frac{2pn}{T} t\right) + b_n \sin\left(\frac{2pn}{T} t\right) \tag{6}$$

$$a_n = \frac{-8V}{n^2 p^2} \sin\left(\frac{npx}{L}\right) \sin\left(\frac{npt}{T}\right) \tag{7}$$

$$b_n = \frac{8V}{n^2 p^2} \sin\left(\frac{npx}{L}\right) \cos\left(\frac{npt}{T}\right) \tag{8}$$

Substituting equation (4) into the Fourier coefficients (7) and (8) and again using the angle addition theorem, we obtain the coefficients a_n and b_n in terms of the ratio (x/L).

$$a_n = \frac{-8V}{n^2 p^2} \sin^2\left(\frac{npx}{L}\right) \tag{9}$$

$$b_n = \frac{4V}{n^2 p^2} \sin\left(\frac{2pnx}{L}\right) \tag{10}$$

From equations (9) and (10) we obtain the coefficients of the magnitude and phase spectra of the displacement of a bowed string undergoing Helmholtz motion as, respectively,

$$s_n = \frac{8V}{n^2 p^2} \left|\sin\left(\frac{npx}{L}\right)\right| \tag{11}$$

and

$$f_n = \frac{npx}{L} - \frac{p}{2}. \tag{12}$$

Equation (11) shows that the magnitude spectrum of the string displacement contains information about the ratio *x/L*. However, for integral values of n, we notice that

$$\sin\left(np\left(\frac{x}{L}\right)\right) = \sin\left(np\left(1 - \frac{x}{L}\right)\right). \tag{13}$$

Interestingly, it would appear to be impossible to distinguish bow direction changes on the basis of the magnitude spectrum, and this result squares with the fact that apart from musical accent and transient perceptual cues during note attacks, it is generally not possible for human listeners to distinguish timbral differences between notes played up-bow and notes played down-bow.

Turning to the phase spectrum, we note from (12) that changes in the ratio (*x*/L) are expected to introduce a linear phase shift, and that (*x*/L) ought to be recoverable from the discrete Fourier transform of measured string displacements by regression of the phase coefficients $\boldsymbol{f_n}$ against the term indices *n* corresponding to the fundamental and higher harmonics of the pitch being played. However, because any displacement of the signal y(t) along the time axis also (by the Time Displacement Theorem) introduces a linear phase shift, this approach to detection of (x/L) is impractical unless the incoming signal is phase locked. In most audio situations, we would prefer to sample the signal without regard to the phase of the waveform. Besides this difficulty, the recovery of (x/L) from the phase spectrum also requires the computational overhead of identifying the relevant Fourier terms (n) for the particular pitch being played, and resolving the issues of spectral leakage that are present in sampled data. We will find that (x/L) is recovered more simply by examining the velocity y′(t) of the bowed string, which may be obtained by differentiating equations (6), (9), and (10) to obtain

$$y'(t) = \sum_{n=1}^{\infty} a'_n \cos\left(\frac{2pn}{T} t\right) + b'_n \sin\left(\frac{2pn}{T} t\right) \tag{14}$$

$$a'_n = \frac{8V}{Tnp} \sin\left(\frac{2pnx}{L}\right) \tag{15}$$

$$b'_n = \frac{16V}{Tnp} \sin^2\left(\frac{npx}{L}\right) \tag{16}$$

These equations describe y′(t) as a square wave (Figure 4) with duty cycle equal to (***t***/*T*). The integral of y′(t) over one period (T) will be zero, and the maximum and minimum amplitudes of y′(t) give velocities *f* and *g* from Figure 1 directly, from

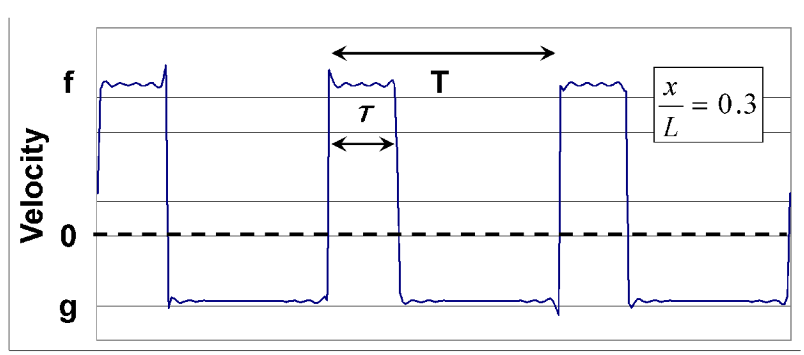

Figure 4: Idealized string velocity versus time (calculated here using the first 10 terms from equations 14, 15, and 16 with x/L = 0.3) is a square wave with duty cycle given by τ/T. Velocities *f* and *g* are indicated on the axis.

which we may estimate (x/L) by equation (1). The velocity versus time signal therefore offers two ways to determine the desired ratio (x/L), both of which are computationally simple compared to analyses in the frequency domain. Measurement of velocity is readily done by the use of conventional magnetic pickups, which respond to the time derivative of magnetic flux through the turns of a coil of fine wire. We expect that the signal from these pickups ought to be proportional to $y'(t)$ since they sense the string directly, without the waveform-distorting compliances associated with sensors positioned at the string-bridge or bridge-body interface. The most reliable determination of bow direction and string length ought to be had when the pickup is located near the bridge, so that (x/L) is minimized and the asymmetry of the waveform (Figure 2) is greatest.

V. EXPERIMENTAL

Data for this study was collected using a violin and magnetic violin pickup constructed by the author. The pickups consist of two coils of #43 magnet wire wound on permanent magnet rods to give each coil a DC resistance of 2200 ohms and an inductance of 0.4 Henry, measured using a Tenma 72-960 LCR meter. The pickup coils were connected in series and phased to effect humbucking. The poles of the magnets were positioned below and between a pair of steel-core strings as described elsewhere [4]. The centers of the magnets were located 1.8 centimeters from the bridge, with the outer diameter of the pickup coils extending to 2.7 cm from the bridge. The overall length of the unstopped strings (L) was 32.5 cm, so that a ratio (*x/L*) of approximately 0.06 to 0.08 would be expected. The pickup output was routed to the input of a Behringer UCG-102 USB audio port, where the AC voltage from the pickup was digitized at 44.1 kHz and 16 bit resolution. This output was recorded using Windows-based recording software (PowerTracks by PG Music). Recorded data was exported to an uncompressed monaural WAV file without altering the sampling rate or bit depth, and then imported into computational software[2] (Octave, version 3.0.1), for the calculations described here. All scripts used are available for download as supplementary files.

We may confirm that the output of the magnetic pickup is closely related to string velocity (Figure 4) by numerical integration of the discrete pickup output signal. Figure 5 shows the results obtained when the trapezoidal method is used to integrate the signal during bowing of the instrument's open E string. The upper and lower plots depict down bow and up bow playing, respectively, and show qualitatively the expected Helmholtz waveform, with the expected reversal of the "sawtooth" wave accompanying change in the bow direction. Apart from experiment, there is no way to discover which waveform correlates with up-bow or down-bow motion, because reversal of the phase of the pickup coil wiring would result in the reversal of the up-bow and down-bow waveforms.

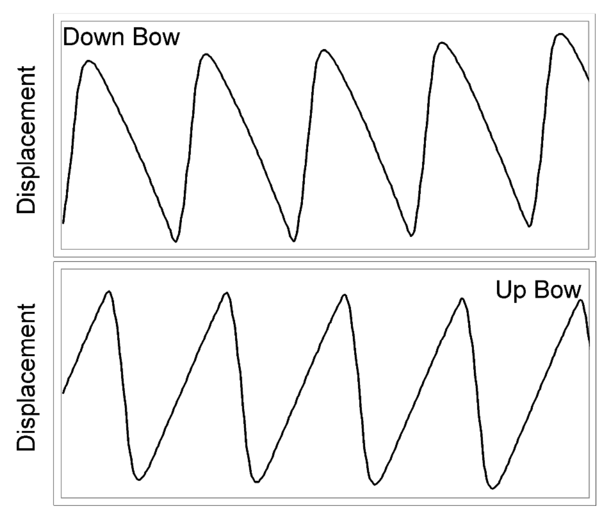

Figure 5—"Displacement" waveforms of violin E string obtained by numerical integration of the pickup signal during up-bow and down-bow playing.

Comparison of these "estimated displacements" with the ideal waveforms (Figure 2) predicted by Helmholtz's equations reveals that the waveforms in Figure 5 show a "rounding off" of the transition from the higher of the two velocities (f) to the slower one (g). When the raw "down-bow" signal from the pickup is plotted versus time without numerical integration (Figure 6), the waveform is seen to be distorted compared to the ideal width-modulated square wave calculated and plotted in Figure 4. In particular, the pickup output waveform is damped so that steep transitions between velocity "f" and "g" are not observed in the output signal. This damping is quantitatively consistent with the L-R time constant introduced by the total 0.8 Henry of inductance and 4.4 kΩ resistance of the series-connected pickup coils. A transient analysis in SPICE (WinSpice 3) was performed using these values for the pickup, in series with an induced voltage source simulated using the waveform of Figure 4. For a series load resistor of 5kΩ, the SPICE model produced a damped waveform very close to that of Figure 6. The SPICE model did not include capacitive

[2] Octave is an open-source GNU-licensed mathematical analysis package that utilizes a scripting language very similar to the MATLAB environment. The author recommends an Octave version that includes a GUI development environment such as GUIOctave by J. Verandas.

elements: it is not known whether the ringing observed in the pickup output is related to self-resonance or to Gibbs phenomena related to bandwidth limiting analogous to truncation of equation (14). In any event, the L-R damping of the pickup introduces systematic error in the measurement of (x/L), but for the purpose of determining bow direction and string length the accuracy obtainable will suffice. An optical pickup capable of sensing the string directly would possibly permit even more accurate analyses by responding more linearly to the string dynamics.

A crude but simple way to estimate (x/L) from the pickup output would be to assume that velocities f and g are proportional to the maxima and minima of the pickup signal over a particular period of time. Somewhat more immunity to noise is had by computation of the average of the maxima and minima of the waveform as shown by the colored horizontal lines in Figure 6. An Octave script ("fng.m", see supplementary files) was written to search for a set of "n" highest-valued (or lowest-valued) data points using maximum likelihood methods. The means of these "n" data points (dashed horizontal lines in Figure 6) were then assumed to be proportional to Helmholtz's velocities *f* and *g*. Using this approach, we estimate the ratio (x/L) for the data in Figure 6 as (0.10 / (0.37 + 0.10)) = 0.21. The solid horizontal lines in Figure 6 represent 99% confidence limits on the assigned values for the maxima and minima.

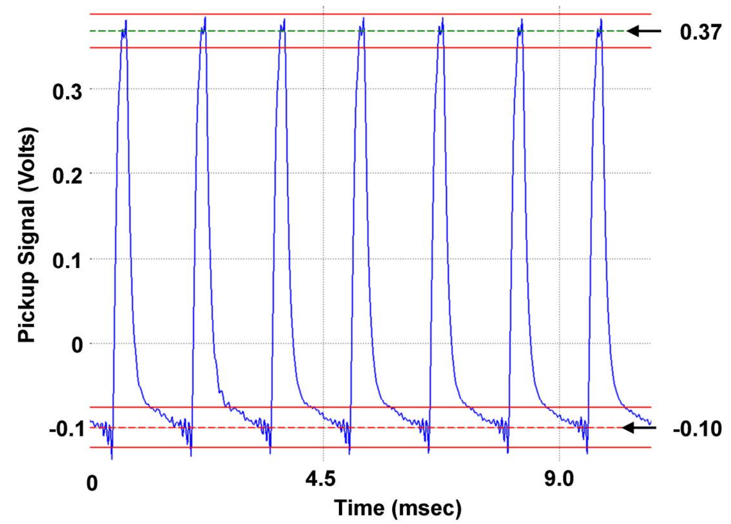

Figure 6— "Down Bow" pickup signal versus time (blue) and estimated values for the maxima (green dashed = 0.37) and minima (red dashed = -0.10) which are proportional to **f** and **g**, respectively (Figure 4). Estimated *x/L* is 0.21 by equations (1) and (4).

The value of 0.21 obtained for (x/L) is larger than the value expected based on actual dimensions of the violin, a systematic error we attribute to distortion of the velocity waveform by inductive damping. Note also that the pickup's effective sensing aperture may not be strictly centered over the magnets: the pickup is responsive to velocity, and the string's peak velocity increases with increasing distance from the bridge (reaching a maximum at x/L = 0.5).

Another way to estimate (x/L) from the pickup output is to estimate the equivalent ratio (***t****/T)* by noting the fraction of one period during which the signal is positive. A second Octave script ("tnt.m" under supplementary files) was written to detect sign changes, and then measure average t and T times for as many whole cycles possible in the data provided, from which (***t****/T)* was calculated directly. The green and red annotations on Figure 7 represent the time assignments made by this script. For the same "up-bow" data the (***t****/T)* estimate was 0.27, which is again larger than expected on the basis of the instrument's dimensions, probably because the L-R damping of the signal artificially lengthens the duration of the positive peaks, introducing systematic overestimation of the time t.

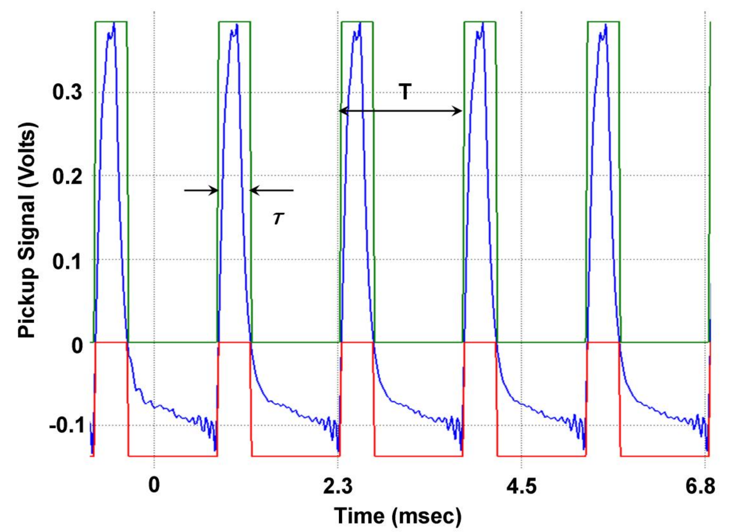

Figure 7—"Down Bow" pickup signal versus time (blue) and results of estimation of values for **t** and T based on elapsed time between sign changes of the waveform. For clarity, only 5 cycles are shown. Average **t** for 9 cycles was 162 samples, average T was 604, yielding an estimated (τ/T) of 0.27 for the audio sample.

We now proceed to the testing of these methods with more complex samples of recorded violin. While the methods shown could be adapted for real-time analysis of (x/L) and (***t**/T*), we will use processing of recorded audio samples to assess how well bow direction and changing string length L is detected. For these calculations, each recorded WAV file was processed, applying the two Octave scripts to consecutive N = 512-point audio segments, to give estimates of (*x/L*) and (***t**/T*) with a time resolution of 12 milliseconds.

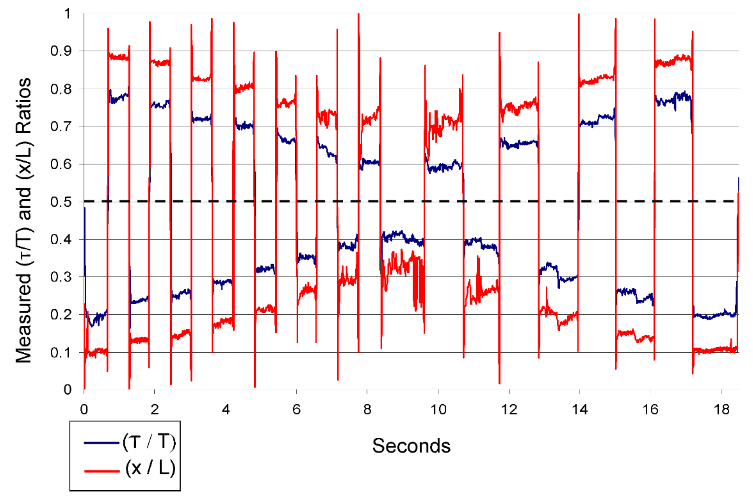

Figure 8—Results of analysis of a two octave E scale played on the E string. The (x/L) ratios more closely approximate values expected from the actual dimensions (x) and (L). However, both (x/L) and (t/T) permit estimation of bow direction and relative string length.

Of course, some segments of audio might represent silence, or might contain a bow direction change so as to contain a mixture of "up-bow" and "down-bow". In a practical implementation of this scheme, provision needs to be made to identify data segments for which reliable analysis could not be performed. One method is to compare the lower and upper bounds on f and g, respectively (lower solid green line and upper solid red line in Figure 6), and reject any data segment for which a clear statistical differentiation between velocities f and g is not obtained.

Figure 8 shows the measured (t/T) and (x/L) ratios resulting from analysis of a two-octave scale played on the E string. Despite the systematic errors just discussed, either ratio permits some assessment of both bow direction and string length. The scale was played using separate bows (beginning with down-bow) per pitch during the ascending part of the scale, and slurring two notes in a bow during the descending portion of the scale. The bow changes are evident in the results as "jumps" in either ratio's value about the dashed centerline of (0.5). In Figure 8, down bow correlates to ratios below 0.5, while ratios above 0.5 indicate up-bow playing. Audio segments during which a bow change was actually in progress produced momentary spikes in (x/L), while the determination of (t/T) ratios was less susceptible to these transition effects. In any event, bow changes were easily resolved with time uncertainties of 25-30 milliseconds.

The results in Figure 8 also demonstrate the anticipated effect of changing string length (L) as the string is shortened by the player's left hand. When bow direction information is removed by subtracting 0.5 from all up-bow notes in Figure 8, we obtain the results shown in Figure 9. The (x/L) ratios (red line) show a trend from 0.1 to 0.3 as the scale ascends. The (t/T) ratios (blue line) also show dependence on string length, and a possibly smaller noise level than the (x/L) ratios estimated from the same data. However, the (t/T) ratios deviate systematically from the (x/L) ratios, which appear to be closer to the calculated (x/L) values from Figure 3 (dark circles in Figure 9).

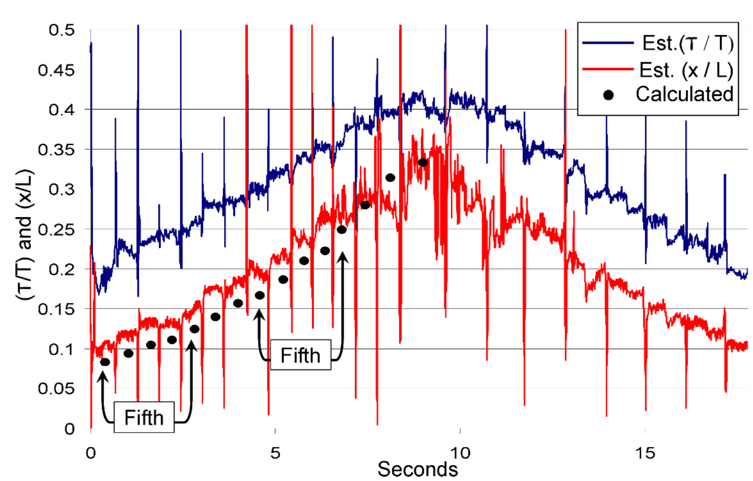

Figure 9 – Measured (t/T) and (x/L) ratios for two-octave scale on the E string after removal of bow direction information by subtraction of 0.5 from all "up-bow" notes. Remaining trend is indicative of string length (L) and permits deduction of player left hand position. Arrows indicate range of a perfect fifth.

During the descending portion of the scale, when two notes are slurred into one bow, neither measured ratio gives adequate string length resolution to reliably detect whole- or half-step changes in pitch. However, if the actual pitch is known by other means (such as a pitch recognition method capable of identifying the fundamental frequency n = 1 in the Fourier expansions above) then the resolution of string length by either (x/L) or (t/T) is still sufficient to deduce the player's string choices. This is illustrated by an example. Considering that the open strings of instruments in the violin family are tuned in perfect fifths, if the (x) and (L) dimensions of the instrument used here are assumed, then the note G5 may be played on the E, A, D, or G strings with (calculated) (x/L) ratios of 0.10, 0.16, 0.23, and 0.35, respectively. Figure 9 shows that either (t/T) or (x/L) estimates still permit empirical assessment of the string length (L) with resolution adequate to resolve changes in the string length associated with notes a perfect fifth apart (indicated by arrows). Thus with knowledge of the pitch being played, it ought to be possible, despite the systematic errors noted above, to deduce which string a player is using on the basis of the measured (x/L) or (t/T) ratios, by empirically relating these values to the position of the player's stopping finger in a manner analogous to the common notation of "fret number" on guitar tablature.

The two humbucking coils of the magnetic pickup used in this study are located between the lowest two (G and D) strings and the upper two strings (A and E) of the violin. A result of this configuration is that during up-bow playing on the uppermost (E) string, the bow hair drives the string *toward* the permanent magnet, increasing flux through the coil. However, during up bow playing on the adjacent (A) string, the bow hair drives the string *away from* the magnet, resulting in a decrease in magnetic flux. Thus if the measured (x/L) or (t/T) ratios are greater than 0.5 for

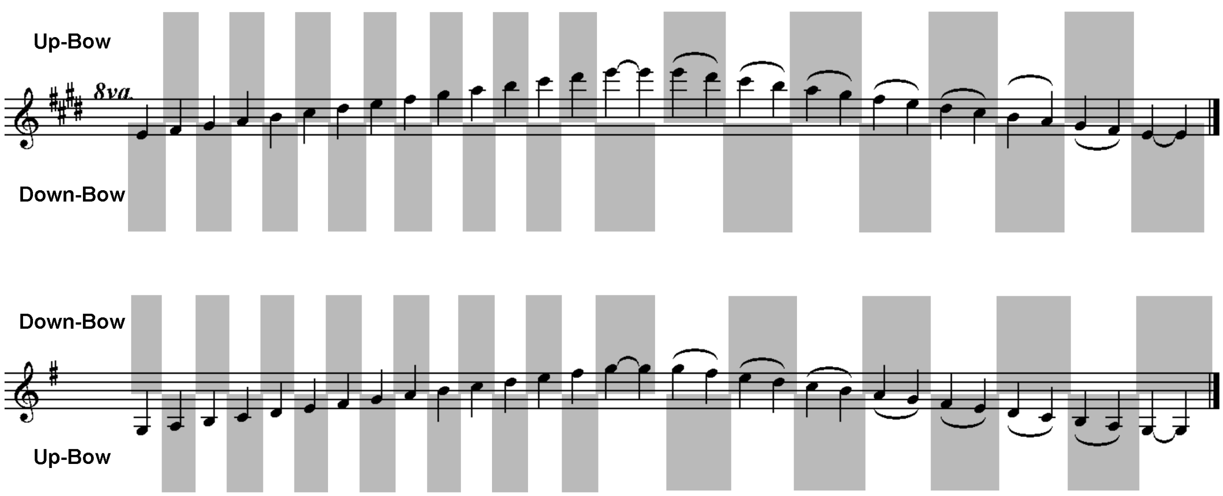

Figure 10: Results of bow direction sensing (shaded bars) for recorded two-octave major scales played on the E string (top staff) and G string (bottom staff) with 12 msec resolution spanning the pitch range of the violin. Bars point upward whenever both (t/T) and (x/L) are estimated to be greater than 0.5, and downward when the ratios are both less than 0.5. The reversal of the down-bow and up-bow ratios between the G and E strings reflects the position of the pickup coils relative to the strings.

up-bow playing on the E string, they are less than 0.5 for up-bow playing on the A string. This effect is seen in Figure 10, which shows bow direction sensing results for two octave scales on the G and E strings, representing the full pitch range of the instrument. During down-bow playing, the bow drives the string *away from* a magnet on the uppermost (E) string, and *toward* a magnet on the lowermost (G) string, with the result that up-bow and down-bow strokes are sensed oppositely on the two strings.

Besides magnet position relative to the strings being sensed, we must consider pickup phasing in the derivation of bow direction. However, these ambiguities can be overcome if the string being used has been identified as described above. Practical implementation of these methods might make use of a simple "training" procedure whereby calibration of the pickup configuration, coil phasing, and open-string (x/L) and (t/T) ratios would be determined for the particular instrument and pickup being used. These steps are not difficult, and a magnetic pickup may be easily added to a conventional acoustic violin, viola, 'cello or bass for purposes of recording the player's bow and left hand technique at the same time the musical performance is recorded by conventional microphones. With an "electric" instrument fitted with a magnetic pickup (either as the main pickup or in addition to piezoelectric sensors on the bridge or body), the bow direction and string length data could be used to trigger different effects or patches during performance, a capability that has been investigated using more complex methods such as instrumented bows[5].

The methods described here are computationally simple and fast enough to follow rapid bow direction changes. Figure 11 shows the results of analysis of the first eight bars of Bach's A minor violin concerto, performed at concert tempo using the three upper strings of the violin. For this analysis, the audio window was reduced to N = 256 points at 44.1 kHz, giving time resolution of bow changes on the order of 12 msec. Estimated (x/L) and (t/T) ratios from the Octave scripts were exported to a spreadsheet, with identification of the player's string choices made by comparing the measured ratios with "calibration" data similar to Figure 9 based on analyses of two-octave scales on all four strings of the test instrument. The bowings used by the player were accurately recovered.

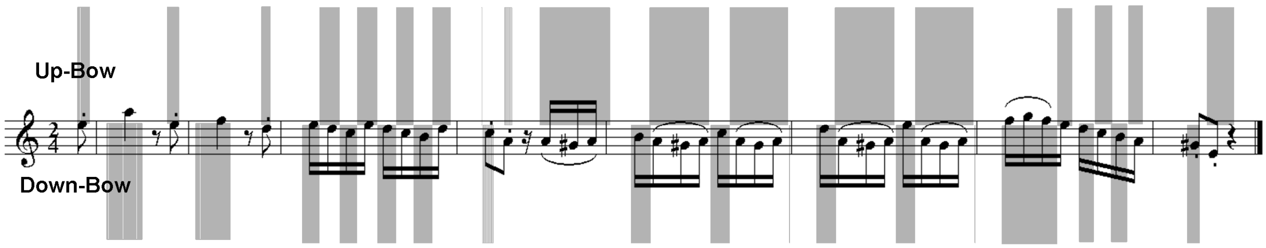

Figure 11—Determination of bow direction for the first eight bars of Bach's Violin Concerto in A minor (BMV 1041) performed at concert tempo. As played, the passage makes use of the D, A and E strings, and required deduction of the string selections by (x/L) and (t/T) ratios, with correction for bow direction based on pickup magnet positions and phasing of the pickup coils in humbucking configuration.

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